DIFFERENT DEFINITIONS FOR SINGLE FLIGHT PROBABILITY OF FAILURE OF AIRFRAMES

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Introduction

Over the years, different ways to calculate the probability of fracture for airframes have been proposed. One specific example is the evolving formulation in the calculation of the single flight probability of failure (SFPoF) by PROF [1,2]. This paper delves into the two different formulae in PROF v2.0 and PROF v3.2, and their applicability to the probabilistic risk analysis of failures at critical locations in aircraft.

For ease of discussion, it should be noted that for two successive events, A and B, the probability of the same outcome is given as:

1) Independent events: $P(A \cap B) = P(A) \cdot P(B)$ (1)

2) Dependent events : $P(A \cap B) = P(A \cup B) - P(A\ only) - P(B\ only)$ (2)

Single Flight Probability of Failure (SFPoF)

Evolution of the SFPoF formulae have been mainly driven by the definitions for survival probability and independent events. In PROF v2.0:

$$SFPoF(t) = \int_0^\infty \int_0^\infty \left(1 - H(a, K_c)\right) g(K_c) f(a)\, da\, dKc \qquad (3)$$

where,

$H(a, K_C)$ is the probability that the crack of size a survives the random maximum stress, given the fracture toughness K_c. The crack size a corresponds to an initial crack of size a_0 at time zero that grew according to the crack growth curve

$f(a)$ is the probability density function of crack size at time, t

$g(K_c)$ is the probability density function of fracture toughness

SFPoF in PROF v2.0 is based on accumulated damage following the crack growth curve and that a structure can only survive the present flight if it has survived the previous flights (i.e., a dependent event). In this instance, $P(B\ only) = 0$ since a structure that did not survive the first event cannot survive the second event. The probability space of B is inside A, and Eq. (2) consequently becomes:

$$P(A \cap B) = P(A \cup B) - P(A\ only)$$

Since *B* is inside *A* then;

$$P(A \cap B) = P(A) - P(A\ only)$$

$$P(A \cap B) = P(B) \qquad (4)$$

Therefore, accumulated crack growth will result in a continuously decreasing survival probability and an increasing probability of failure in later flights.

In PROF v3.2, a new calculation for SFPoF using the Freudenthal method has been proposed (see Eq. (5)) [1,2].

$$SFPoF(t) = \int_0^\infty \int_0^\infty \left(1 - H\left(\frac{K_c}{\alpha(a_0, t)}\right)\right) \prod_{i=1}^{i=t-1} H\left(\frac{K_c}{\alpha(a_0, i)}\right)\ g(K_c) f(a_0)\, da_0\, dKc \qquad (5)$$

Eq (5) is based on discretising the survival probability per flight hour and that for a crack to fail at t^{th} flight hour it has survived until "t-1" flight hours as indicated by the product of the survival probabilities until "t-1" and multiplying the failure probability at t flight hours.

Comparison of Equations (3) and (5)

To compare Eq. (3) and Eq. (5), hypothetical input parameters were used to calculate the SFPoF's:

Crack growth curve : $a(t) = a_0 e^{\nu t}$

Maximum stress (Gumbel) : $F(x) = e^{-e^{-(x-10)/1.27}}$

EIFS (Lognormal) : μ the mean of $\ln(a_0)$ σ the st. dev of $\ln(a_0)$=0.5

Fracture toughness : $K_c = 32$

Beta factor assumed as 1.0 for all cracks

For Eq. (3) and Eq. (5), SFPoF was plotted against time (Figs 1 and 2) to show the sensitivity of the SFPoF curve to crack growth rate, ν, and to the mean of EIFS, μ .

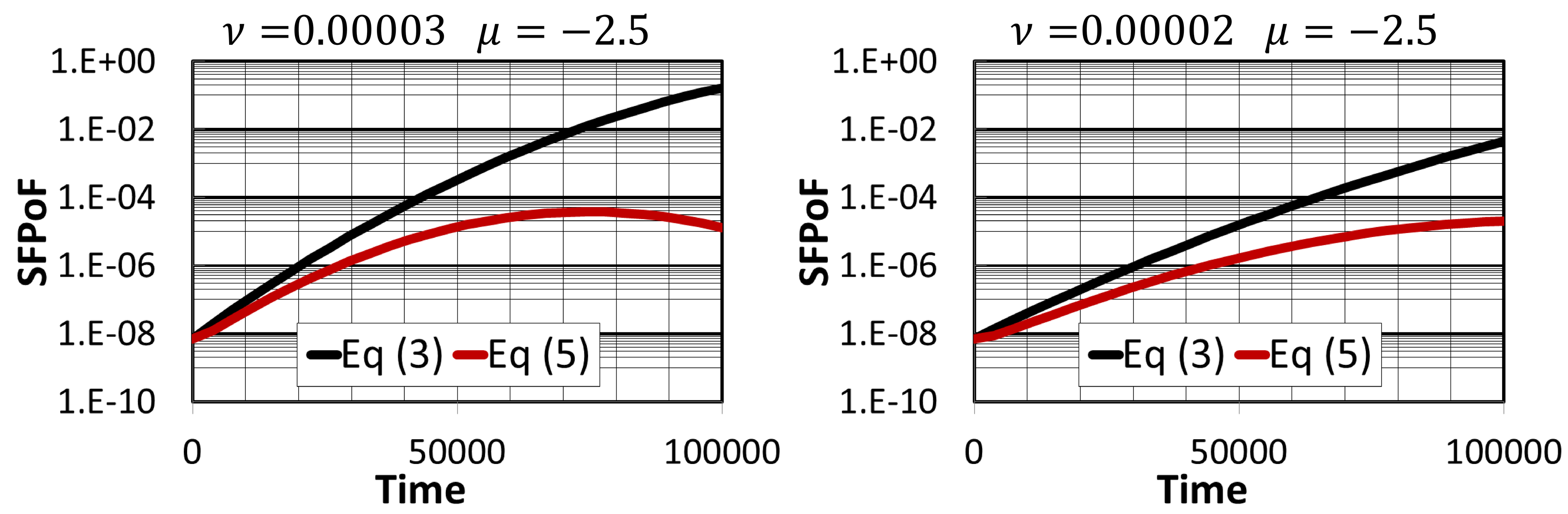

Fig. 1 Sensitivity of SFPoF curve to the crack growth rate, ν

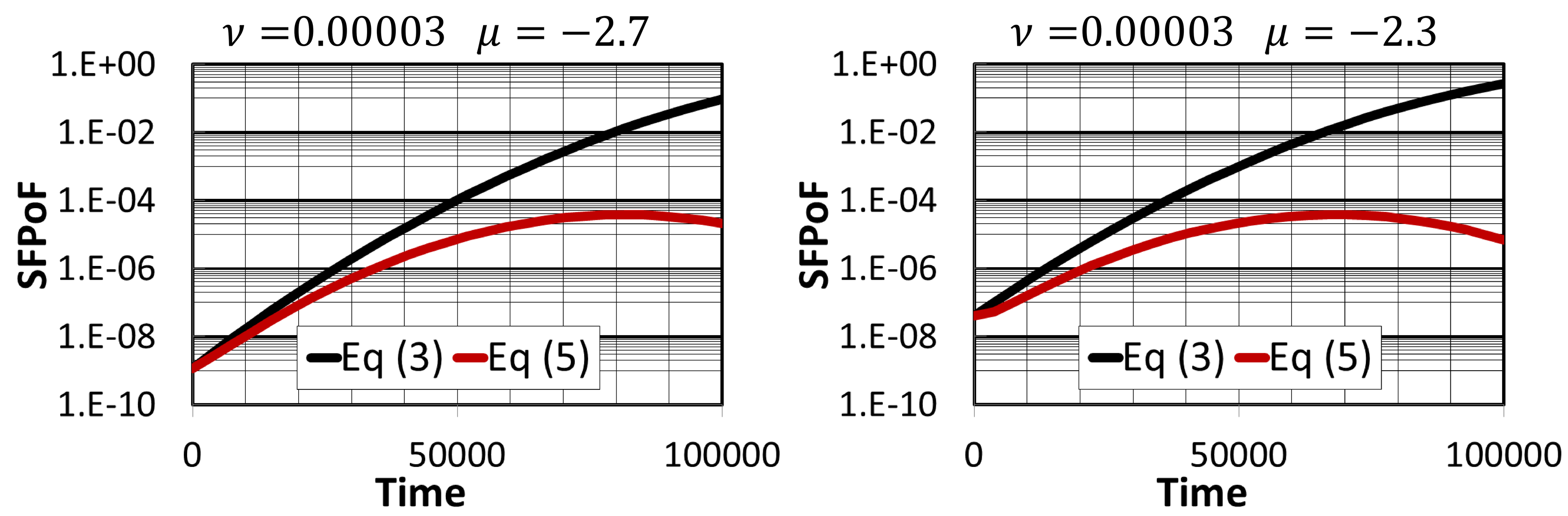

Fig. 2 Sensitivity of SFPoF curve to the EIFS mean, μ

To be able to multiply the succesive probabilities of survival , $\prod_{i=1}^{i=t-1} H\left(\frac{K_c}{\alpha(a_0,t)}\right)$ in Eq. (5), events must be independent of each other (Eq. 1). However, with the crack growth curve as an input, the probability of survival in each flight is conditional on non-failure in all previous flight hours. To satisfy the independence requirement, it must be assumed that a crack is not growing (i.e., ν =0.00) (Fig 3). This assumption indicates that SFPoF in each flight hour is solely dependent on the random maximum stress per fllight hours and independent of previous flights.

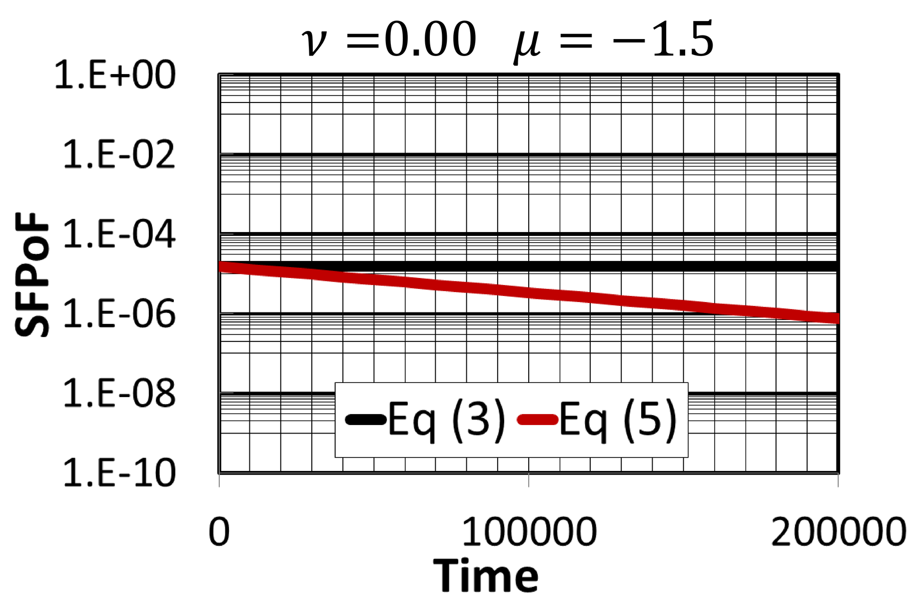

Fig. 3 SFPoF of non-growing cracks

Comments

- Probability of failure during the next flight hour is defined in Eq. (3). For a scenario when a crack is no longer growing, there is a constant probability of failure.
- Probability of failure during a specified flight hour on the condition that each successive events are independent is defined in Eq. (5). For a scenario when a crack is no longer growing, there is a decrease in SFPoF with time.
- The sum of the SFPoFs from $t = 1$ to $t = \infty$ in Eq. (5) is equal to 1.0 assuming all probabilities are independent. However for conditional probabilities, the sum of all probabilities from $t = 1$ to $t = \infty$ may not add up to 1.0. This is inconsistent with probability theory for an event that is certain to occur with time.
- In contrast to the risk curve produced with Eq. (3), the risk curve produced with Eq. (5) is atypical and less conservative (Figs 1 - 2).
- Eq. (3) can be used to predict how long it is safe to fly before inspection given an acceptable level of risk. Eq. (5) can be used to predict the statistics or distribution of discrete probabilities (i.e., probability mass function) in time.

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[2] Brussat, T., (2012) Recommended Methodology Updates to Improve Single Flight Probability of Failure Estimation, ASIP2012