Australian Government
Department of Defence
Science and Technology

A bounded distribution model of equivalent initial flaw size for structural risk analysis

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10th Structural Integrity and Failure Conference Adelaide, 12-15 July 2016

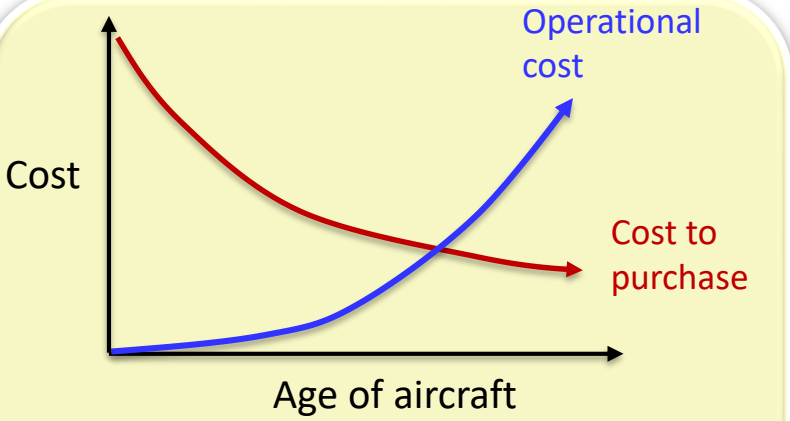
Outline of presentation

- Background on probabilistic approach to failure assessment
- Advantages of probabilistic approach
- Equivalent initial flaw size modelling
- Results and discussion
- Conclusion

Fatigue failure risk analysis – what it brings to Defence

- Enhance safety
- Maximise availability
- Minimise cost of ownership

Cost of ownership of military aircraft



AIRCRAFT	MISSION	COST TO FLY PER HOUR	COST TO PURCHASE
VC-25	Air Force One	$206,337	$330M*
AC-130H	Special Operations	$173,253	$210M
E-4B	National Airborne Operations Center	$149,580	$223M
B-2A	Bomber	$128,805	$1.16B**
E-8C	JSTARS	$73,234	$244M
B-52H	Bomber	$67,005	$84M***
CV-22B	Special Ops	$65,684	$90M
F-22A	Fighter	$59,166	$143M
B-1B	Bomber	$58,488	$317M
F-35A	Fighter	$42,169	$108M ****

*Fiscal 1990 dollars **Fiscal 1998 constant dollars ***Fiscal 2012 constant dollars **** Most recent price from Lot 8 contract

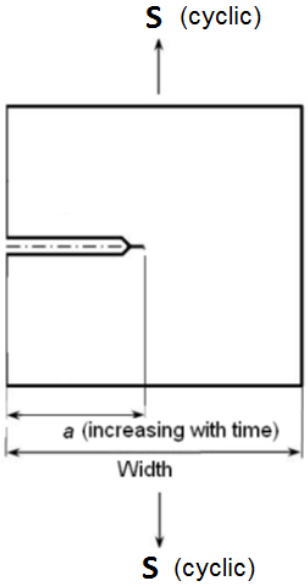
Source: Air Force Reporting: Phillip Swarts/Staff and Lara Seligman/Staff

Fatigue failure risk analysis :

- Operational life of an aircraft
- Safety inspection interval

US Data

When does fatigue failure occur?



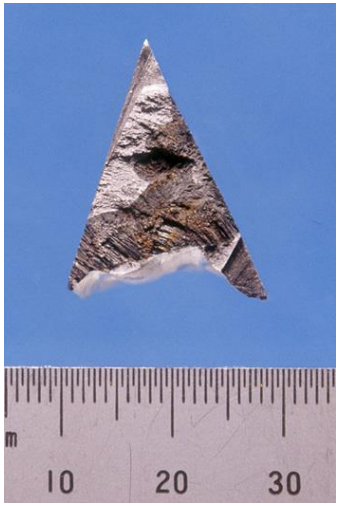
Fatigue failure occurs when :

$$K_C \leq S \cdot \beta(a)\sqrt{\pi a}$$

or

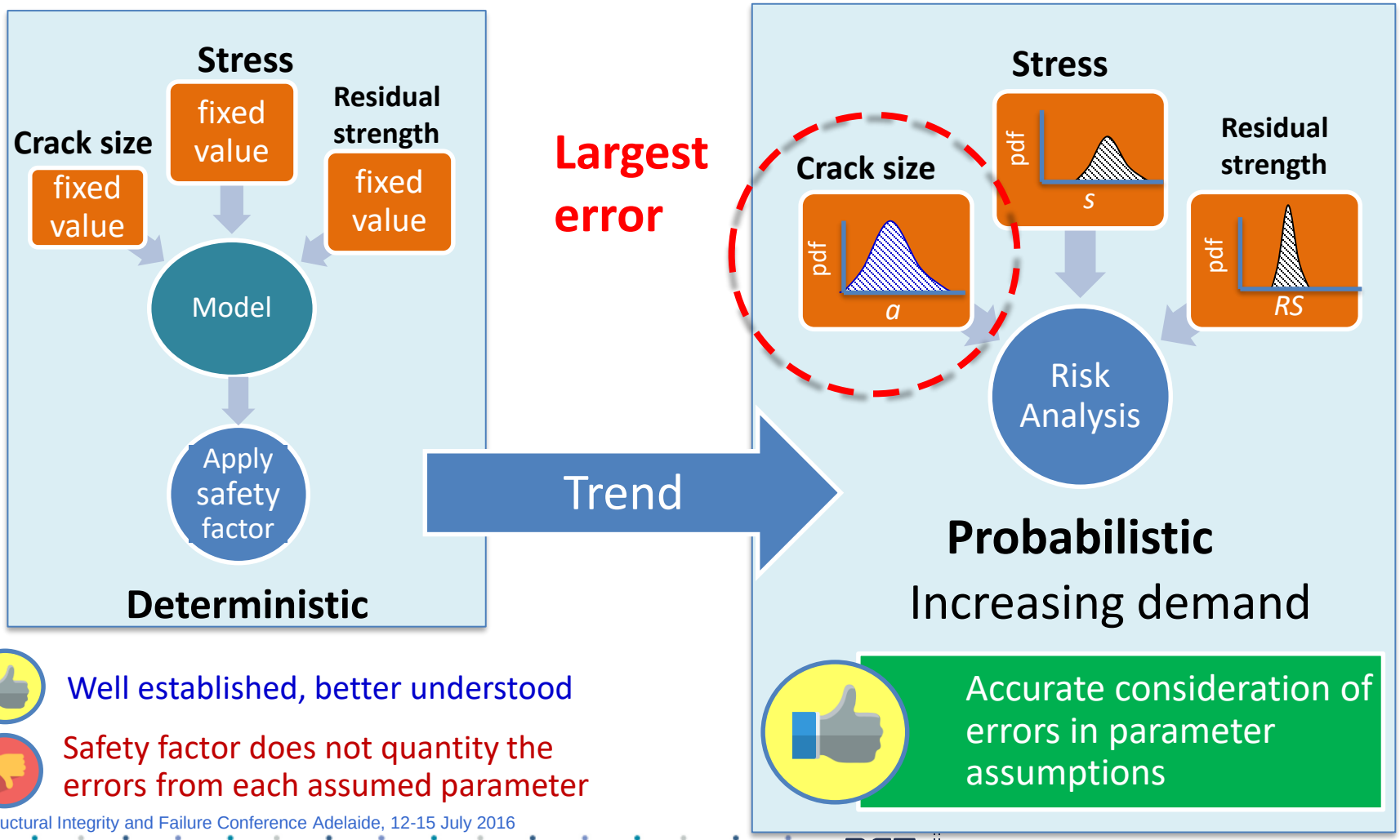
$$S > Residual\ Strength$$

Kc : stress intensity factor
S : cyclic stress applied
A: crack size
β(*a*) : geometry correction factor



Global trend towards probabilistic approach

"Those who will begin with certainties, shall end in doubts;
but those who will be content to begin with doubts, shall end in certainty" - **Francis Bacon**



Well established, better understood

Safety factor does not quantity the errors from each assumed parameter

Obstacles in probabilistic analysis of failure

High sensitivity to input data

- **Initial crack size distribution**
- Variable stresses, material properties, etc.

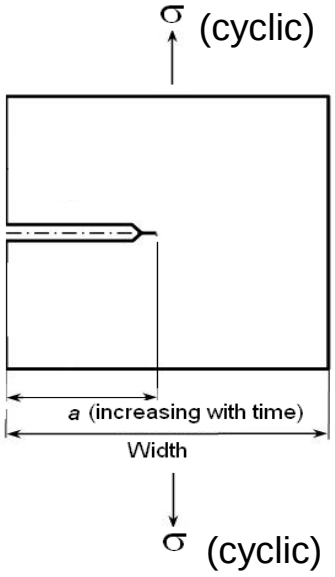
Lack of accuracy in models representing the data

- Lognormal distribution, Weibull distribution, etc.

Lack of input data

- Prohibitive cost in obtaining data
- Location specific

Probability of Failure



- Risk - probability of failure or unstable fracture
- Failure occurs when; $\sigma \geq \text{Residual strength}$

Probability of Failure (PoF) calculation:

$$PoF = \int_0^{\infty} \boldsymbol{f(a)} \left(1 - \int_0^{S_{RS}(a_{cr})} f(s)\, ds \right) da$$

Where :

s = stress

a = crack size

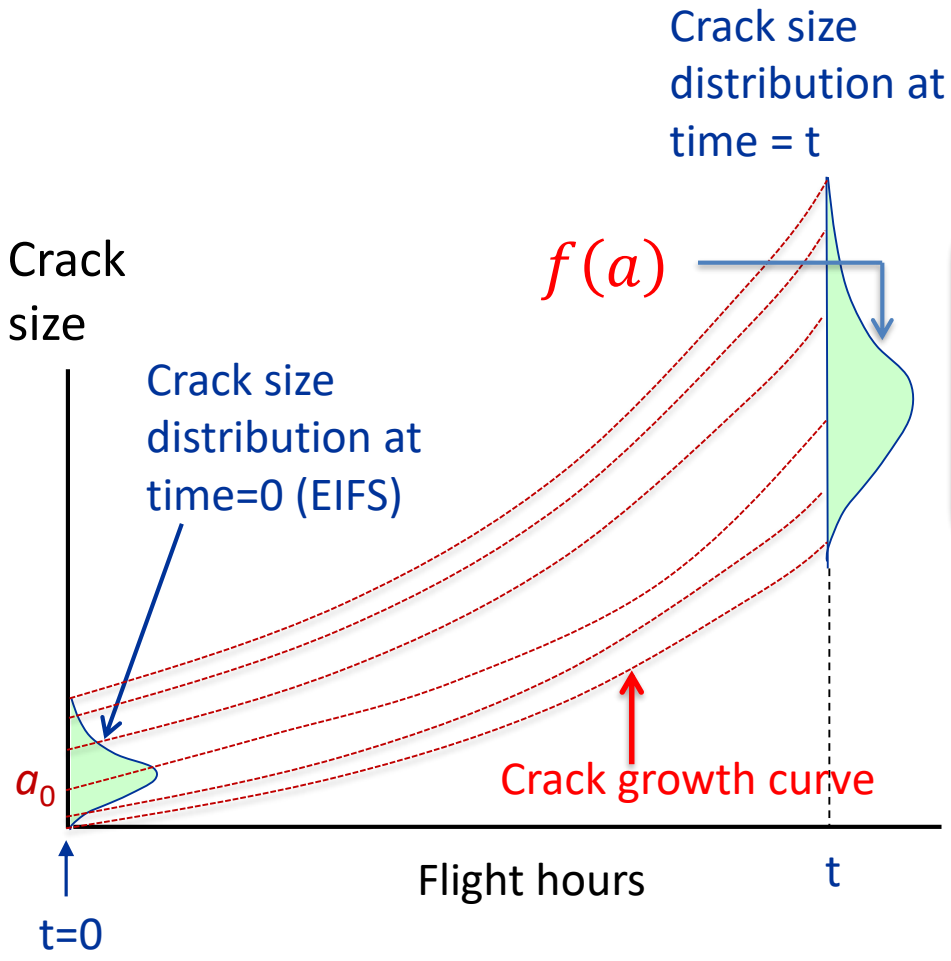
a_{cr}= critical crack size

s_{RS}= residual strength

$f(a)$= crack size probability density function

f(s)= stress probability density function

Great uncertainty in crack size (a) prediction



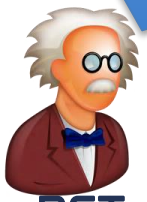
Fracture occurs when:

$$K_C \leq S \cdot \beta(a)\sqrt{\pi a}$$

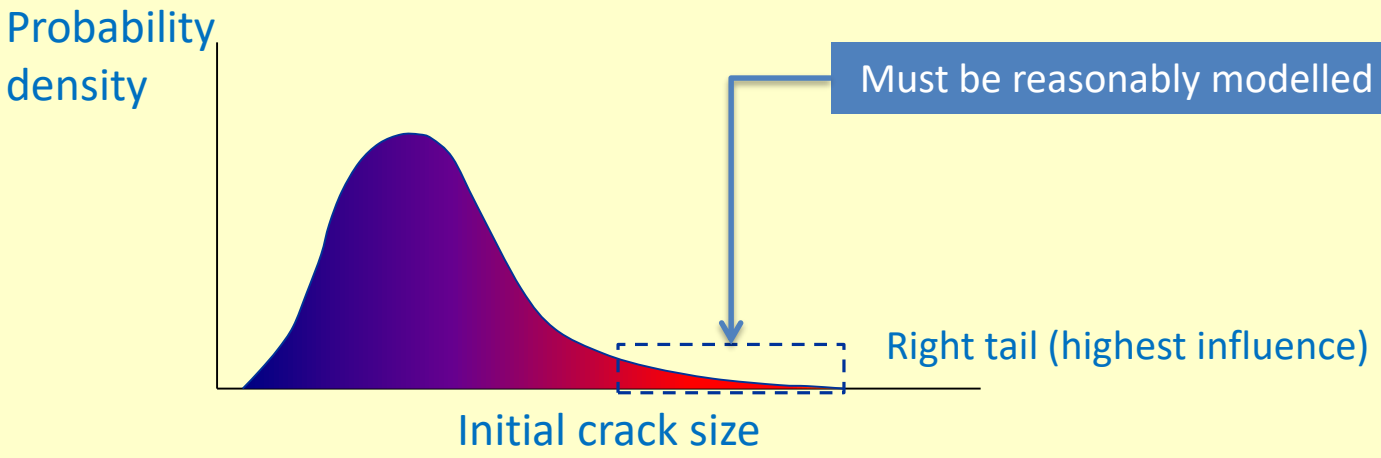
Predicting crack size??

- Uncertain initial crack size
- Crack growth modelling inaccuracy
- Material property not uniform
- Loads fluctuate

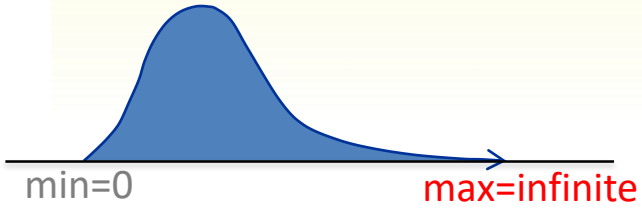
Predicting crack size variability is more practical than predicting crack size



Right tail of distribution – critical for PoF prediction

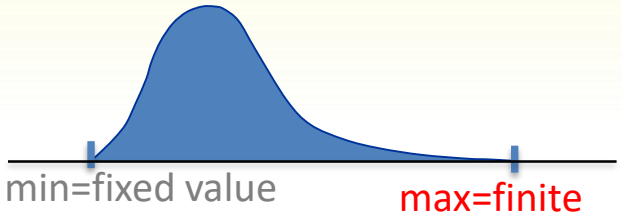


Lognormal distribution



Conventional model

Beta distribution

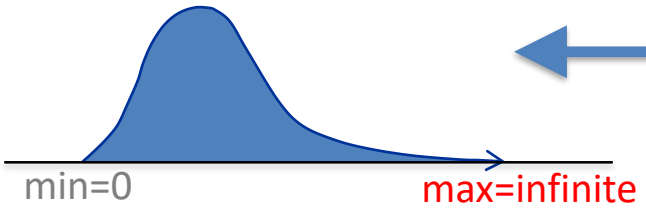


More realistic model

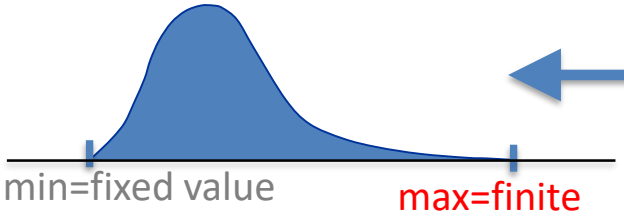
Derivation of EIFS distribution (Direct method)

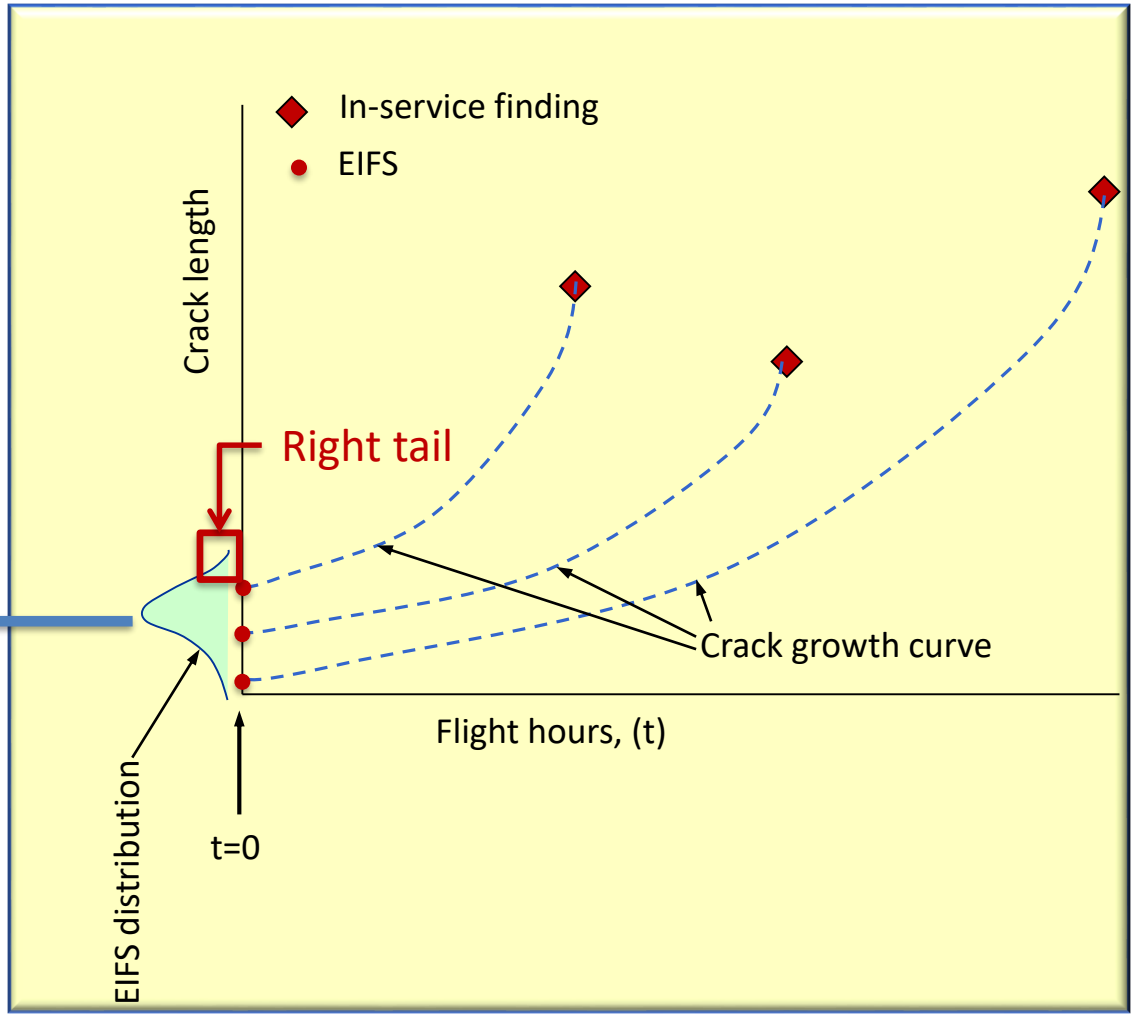
Distribution models used:

1. **Lognormal**

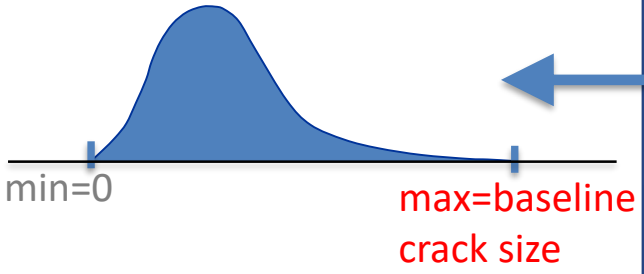


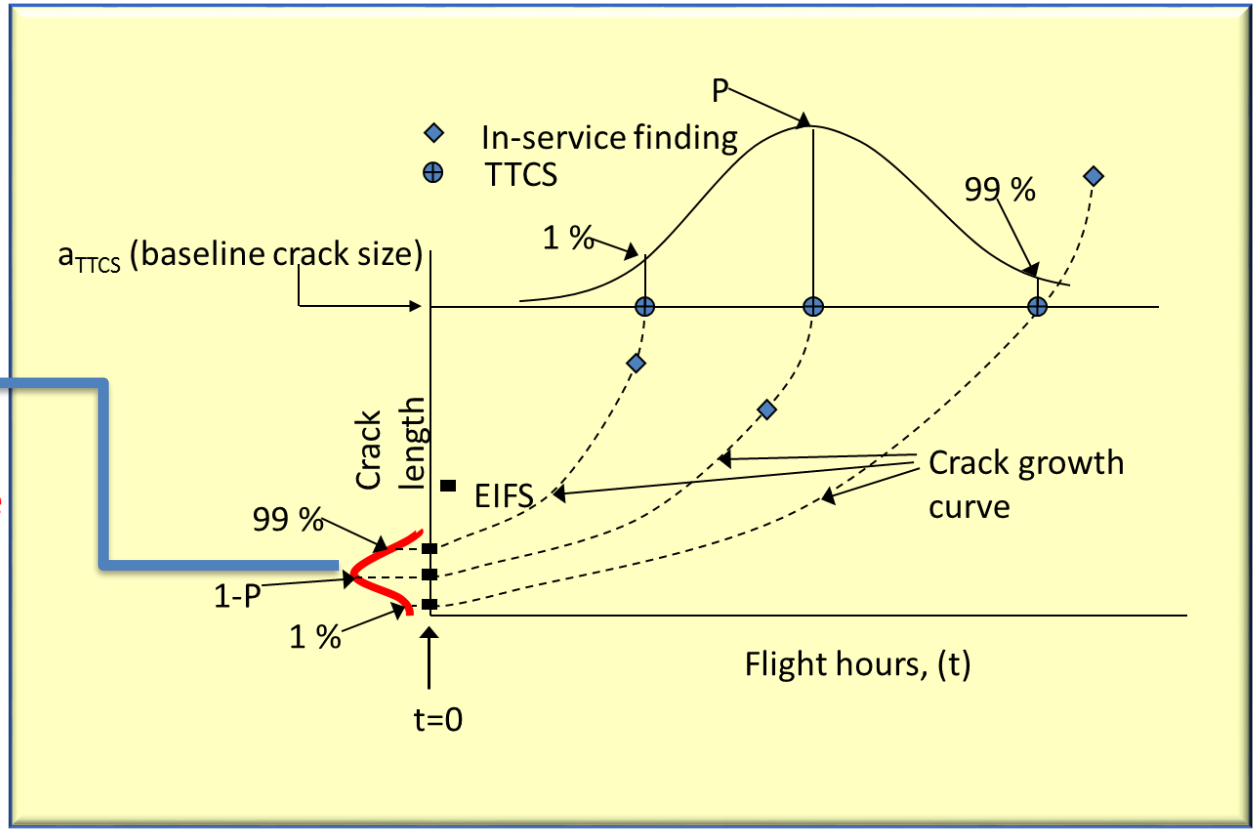
2. **Beta**





Derivation of EIFS distribution (TTCS method)





Teardown inspection crack data

Total raw data = 145

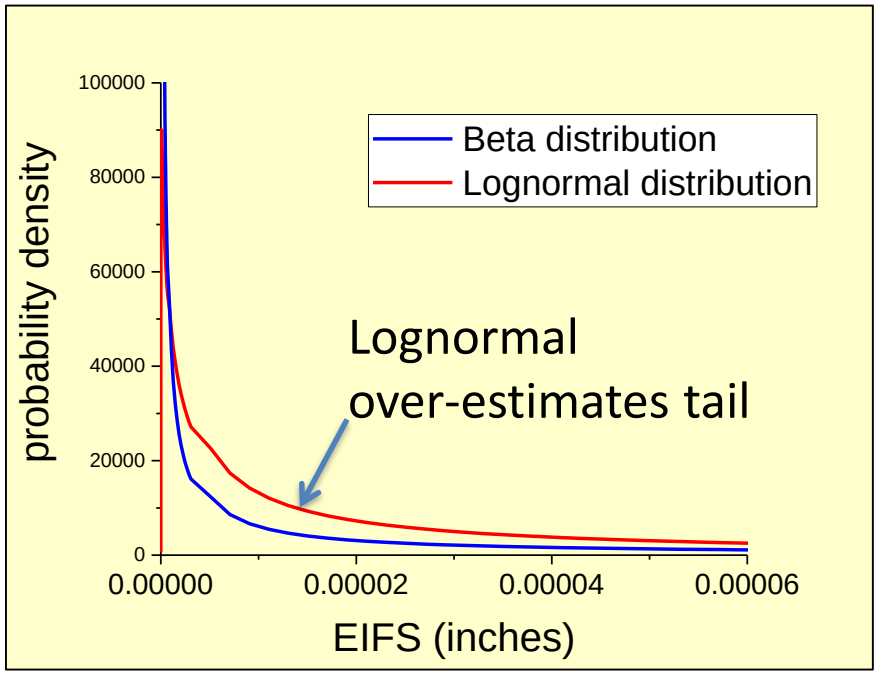
Filtered for RAAF or USAF Fleet
No. of data = 100

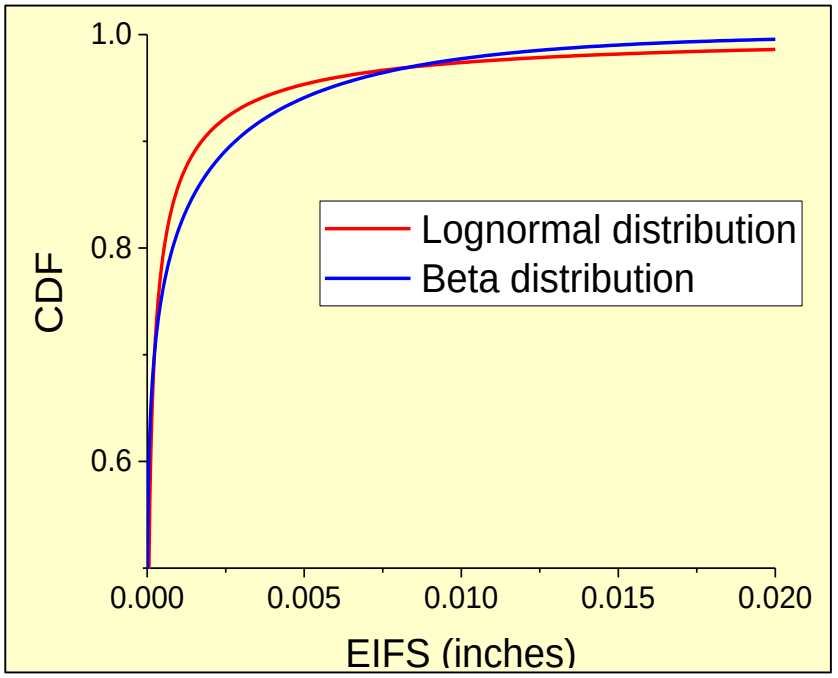
Filtered for non-MSD data
No. of data = 65

Data used for regression = 65

Initial crack size data (EIFS) modelling

Lognormal distribution vs Beta distribution

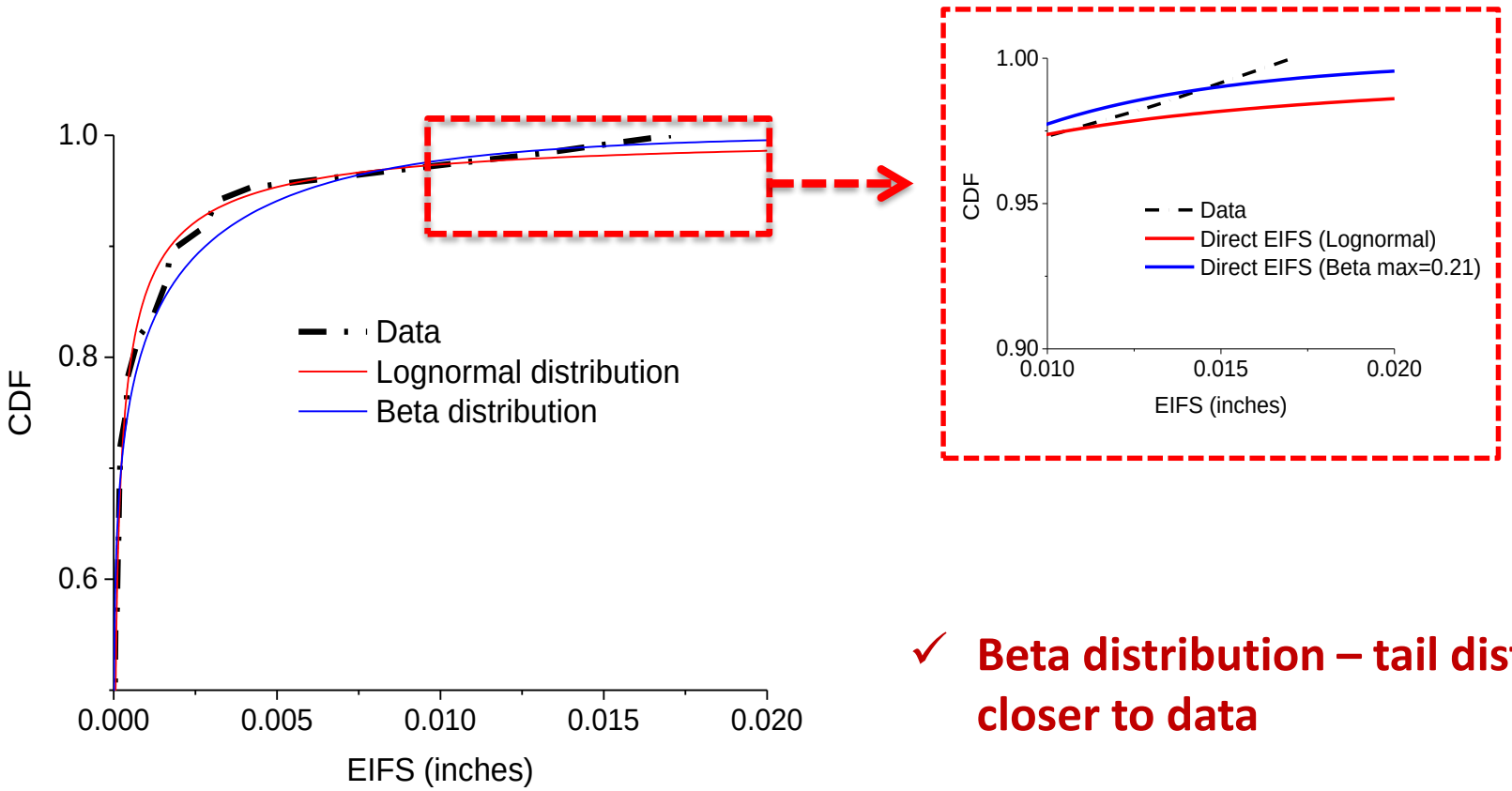




Probability density

Cumulative probability

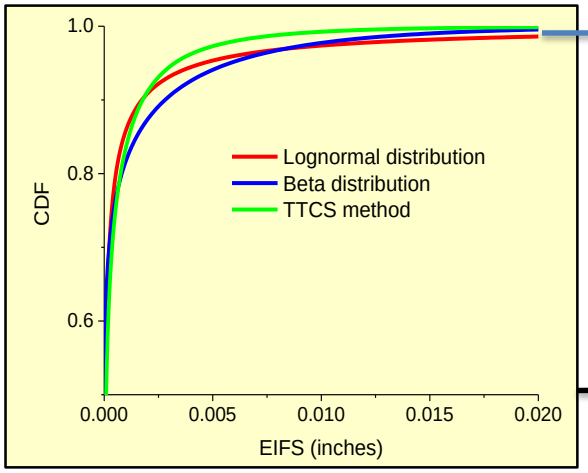
Distribution model goodness of fit

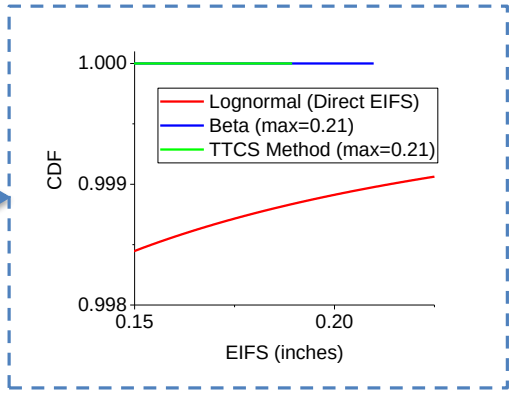


✓ **Beta distribution – tail distribution closer to data**

EIFS Distribution and corresponding Probability of Failure, POF

EIFS distribution, $f(a)$

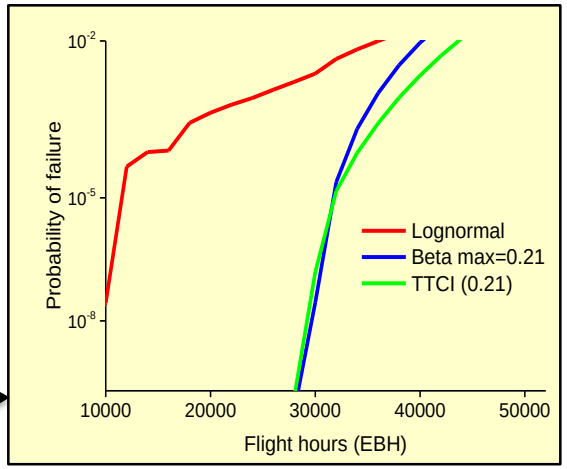




$$PoF = \int_0^{\infty} f(a)\left(1 - \int_0^{S_{RS}(a_{cr})} f(s)ds\right)$$

POF calculation

POF curves



for CDF=0 to 0.97 (i.e., 97%)

- Lognormal dist. in between Beta and TTCS
- The three models are very close until CDF=0.8

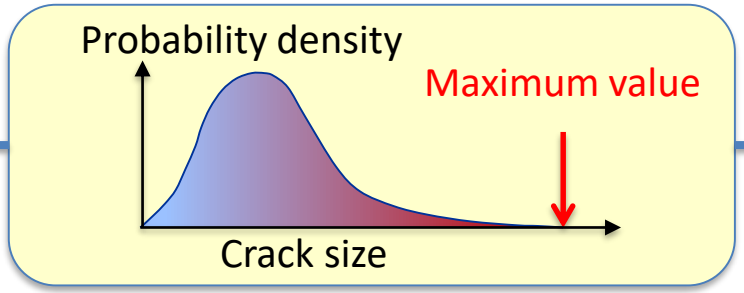
- Beta and TTCS methods much closer
- Lognormal way higher

from CDF=0.97 (i.e., only 3%)

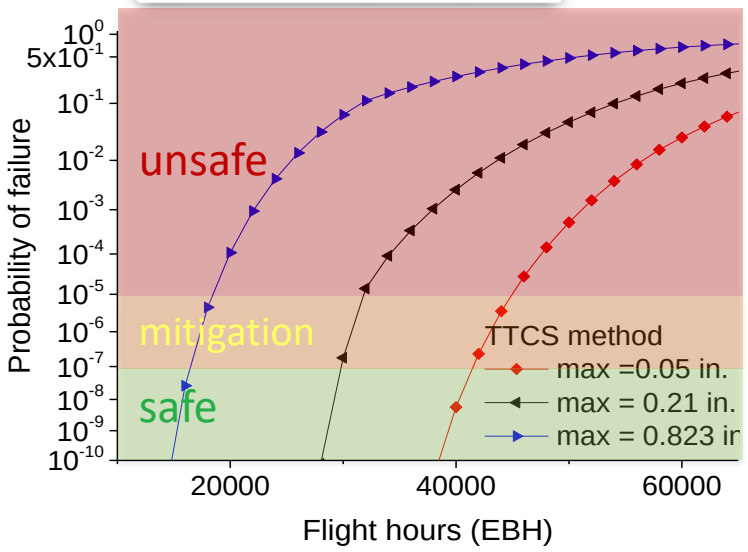
- Beta and TTCS methods closer

Lognormal distribution's unbounded maximum value overestimates risk by a large margin !

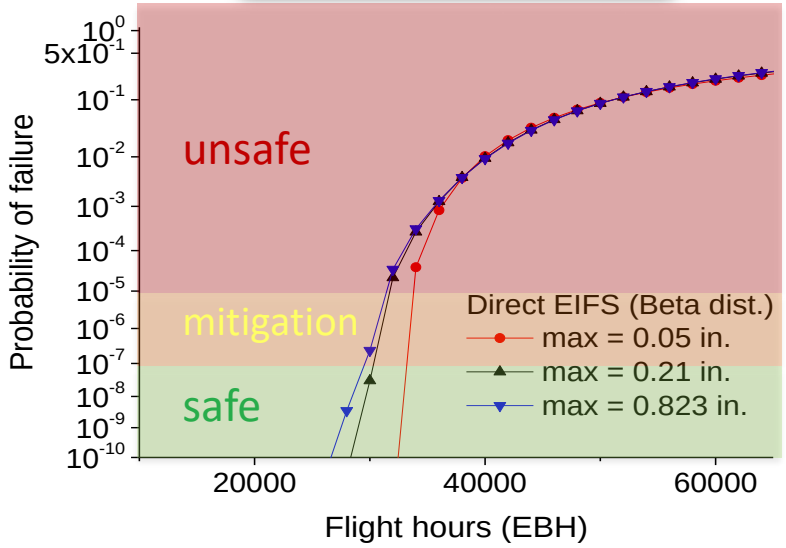
Sensitivity of risk values to maximum values of the distribution



TTCS method



Beta distribution



Highly sensitive to assumed maximum value

Less sensitive to assumed maximum value

Conclusion:

1. Direct EIFS beta distribution model is superior to TTCS method in fitting its model to the data or regressed values at EIFS level.
2. Beta distribution shows better goodness of fit to the observed or regressed EIFS values than the lognormal distribution.
3. Probability of failure (PoF) curves from TTCS method are very sensitive to the assumed maximum EIFS.
4. Overall the using Beta Distribution seems to be a better option compared to the other two models since it showed superior results in terms of desirable characteristics of an EIFS distribution model.

Future work:

- Investigate the use of extreme value distribution for EIFS modelling
- Investigate the sensitivity of risk predictions from other input parameters (e.g., stress distributions, material properties)

Questions?